REMR Technical Note CS-ES-2.5

ECONOMICALLY OPTIMAL NONDESTRUCTIVE EVALUATIONS OF STEEL STRUCTURES

PURPOSE: To demonstrate a method for optimal selection of nondestructive evaluation (NDE) to determine the condition of steel structures.

BACKGROUND: Technical Note CS-ES-2.4 deals with reliability estimations of structural features common to Corps of Engineers Civil Works steel structures. These estimations can provide useful information as to the expected utility and safety of structures for the conditions known or assumed in the calculations. Although reliability estimations are useful in themselves, in engineering practice these analyses are usually performed in situations in which the engineer must make a decision. This decision may involve determining whether the structure is adequate, should be repaired, or even replaced. The consequences of this decision may depend on some factor that is not known with certainty, a factor called the "state of nature." Recognizing that the uncertainty of what the true state of nature is can be expressed as probabilities, the engineer can analyze the alternative decisions to determine the optimal choice. Where uncertainty exists regarding the state of nature, it is often possible to obtain more information by nondestructive testing. Because there is often some uncertainty in obtaining and interpreting these test results, the need for a probabilistic approach for incorporating these results into the decision-making process is indicated.

One probabilistic approach for decision making that recognizes the subjective elements as well as the objective aspects of the analysis is the Bayesian Statistical Decision Theory. This theory derives its name from Thomas Bayes, a mathematician who introduced the equation now used to relate certain probabilities in the decision model. A detailed description of the Bayesian Decision Analysis is provided in Ref a, and its application to civil structures is described in Ref b.

Basic components of the "prior" decision model (before any testing) are illustrated in Figure 1. The engineer can choose between a number of alternative actions a_1, a_2, a_3,,,,a_n in the action space "A." Once the decision has been made, the engineer must wait to see which of the possible states of nature, θ , is the true one. As a result of taking this action and finding the true state of nature, the engineer will receive value or utility, (U) (for example, dollars) of the consequences. This decision model may be illustrated by the example shown in Figure 2 for deterioration of sheet piling. At any time in the life of the structure, the engineer may be required to decide what, if any, action should be taken. As shown in the decision tree, the engineer may have to decide whether the sheet piling should be replaced. After the engineer makes a decision and takes an action, the true state of nature will be found; i.e., either the piling will remain functional for its intended life or it will not. The utility received can be considered to be the cost (negative utility) of the action and consequences. It should be noted that utilities

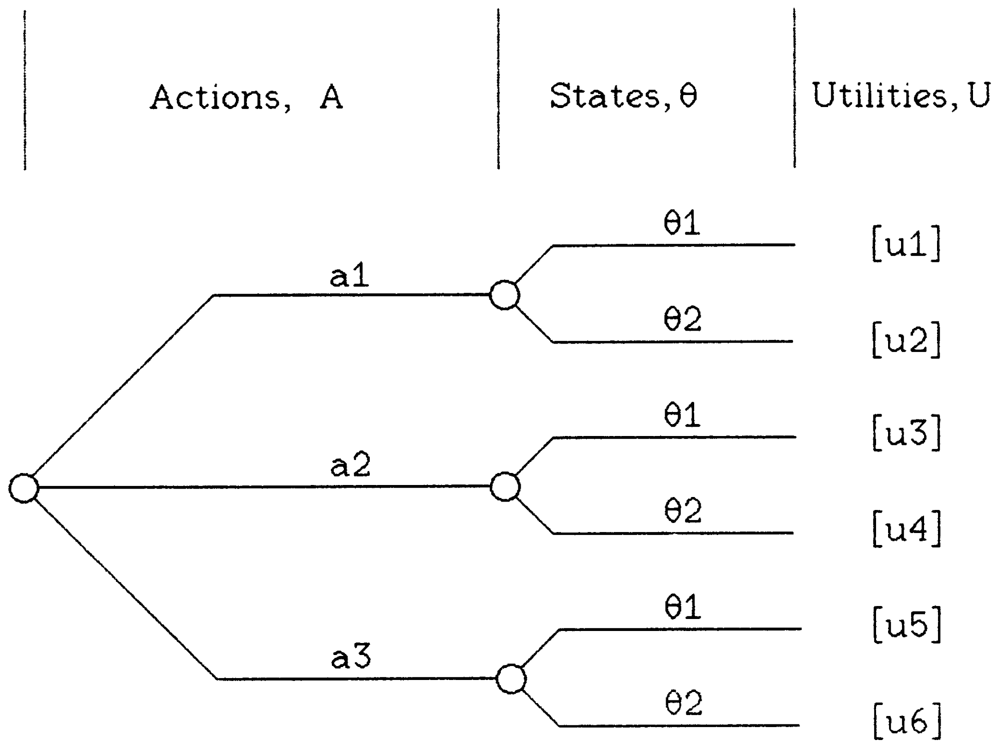

Figure 1. "Prior" decision model

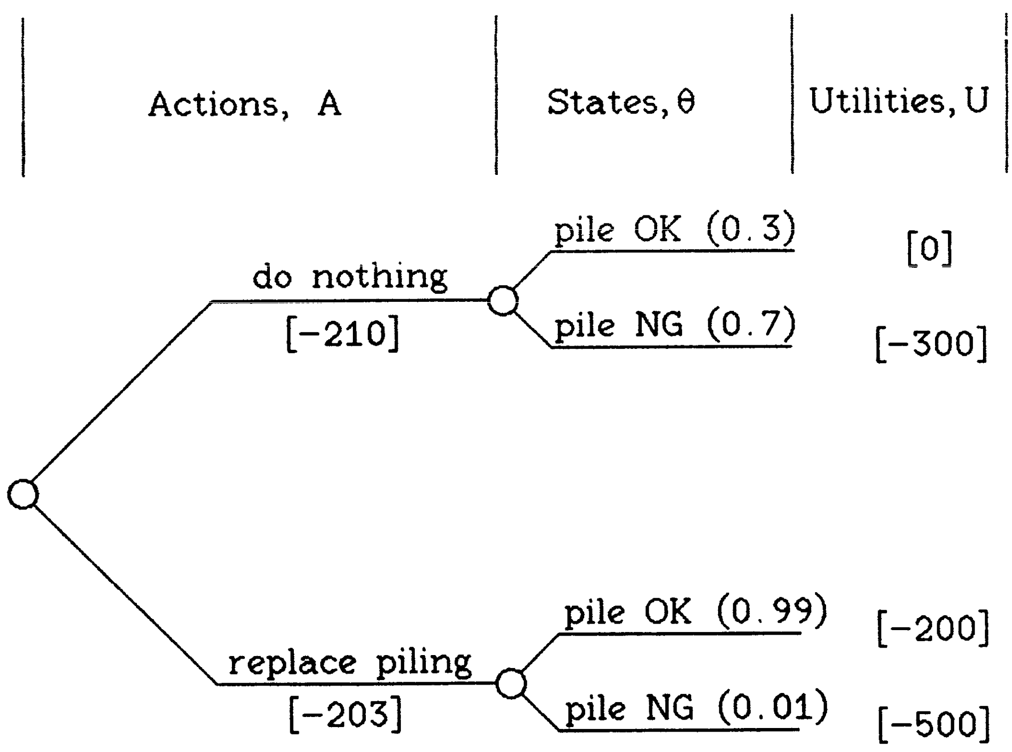

Figure 2. Example sheet pile decision model

need only be relative; they do not have to represent actual costs. For the example shown, the engineer can determine probabilities associated with the possible consequences by performing a reliability estimation. A "prior" estimate of the corrosion from past experience (or any other guidance) projected over the life of the structure will indicate the probability of the structure's functioning.

In the example shown (Figure 2), the engineer has determined only 30-percent probability that the structure will remain functional if no action is taken; of course, there will be no associated cost. Along with the 70-percent probability that the sheet pile will not function if nothing is done is a utility of -300, representing a relative cost of replacement and losses associated with the loss of function. If the structure is rehabilitated at a relative utility of -200, the engineer has determined that the reliability is now 99 percent. The 1-percent probability of functional loss incurs a utility of -500, which represents the initial cost of rehabilitation (-200) and the costs of replacement and functional loss (-300). By multiplying the related probabilities with their utilities, the expected cost of each action is computed as shown; i.e., a utility of -203 for rehabilitation and -210 if nothing is done. Based on this decision analysis, rehabilitation is the optimal action choice.

The engineer has another choice not shown in this decision tree. He usually has the option to "buy" better information by performing tests to improve the accuracy of the assumed probabilities. This better information has an associated cost, which directly affects the utilities of each consequence. The costs and probabilities of the tests' predicting the true state of nature can be incorporated into the decision process by an adaptation called Preposterior Bayesian Decision Analysis. ("Posterior" refers to analysis incorporating known test results and "pre" refers to projecting the analysis before the tests are done.) The probabilities determined from the prior decision analysis are modified by those resulting from new information. Thus, all of the available information, old and new, is retained in the posterior analysis, weighted by its relative uncertainty.

As shown in Figure 3, the decision model now includes two additional factors, the experiments (tests) and possibe outcomes of the experiments. As the result of an experiment, e, the engineer observes new information (outcome) z, in the space of all possible outcomes, Z. This new information is combined with the prior probabilities to obtain posterior probabilities using Bayes' rule, which can be stated as:

$$P''(\theta_i) = \frac{P[z_k|\theta_i]\ P'[\theta_i]}{\Sigma_j\ P[z_k|\theta_j]\ P'[\theta_j]} \qquad (1)$$

where

$P''(\theta_i)$ = posterior probability of θ_i given sample outcome z_k

$P[z_k|\theta_i]$ = probability of observing outcome z_k as a function of the true state of nature θ_i (likelihood)

$P'[\theta_i]$ = prior probability of θ_i

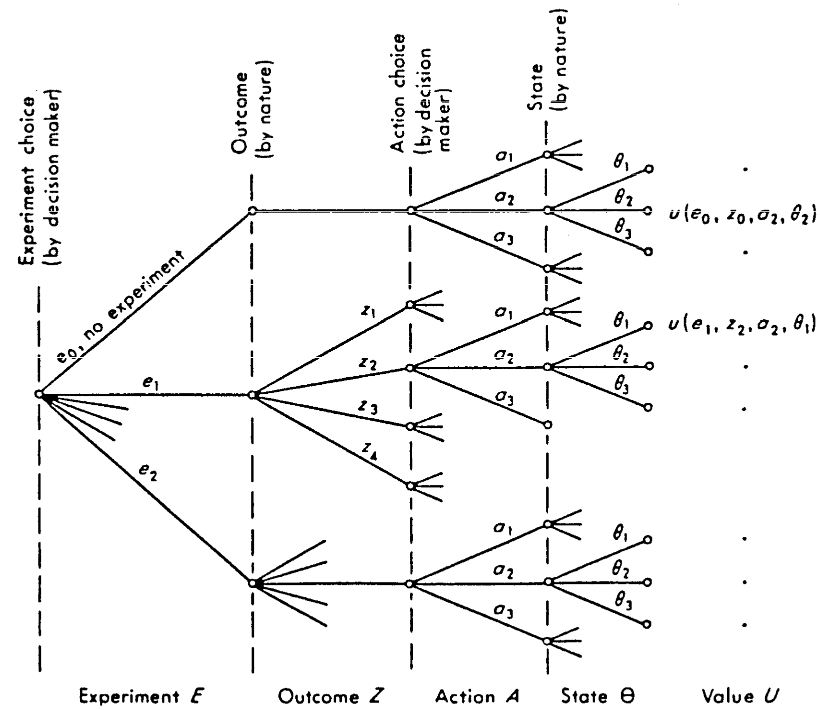

Figure 3. Preposterior Bayesian Decision Analysis (Ref a)

The denominator represents a normalizing term to ensure an area of unity for the function $P''(\theta_i)$. The probabilities for the various states and the possible experiment outcomes are modified by the above procedure. Modified probabilities are then used to compute the expected costs.

Preposterior Bayesian Decision Analysis can be a useful management tool to determine an optimal course of action when a decision regarding NDE is required. The procedure is tutorially illustrated by application to the three structural features analyzed in the preceding section. For the purposes of this illustration, certain assumptions have been made regarding "prior" conditions and utilities (costs). The assumed values are based on discussions with Corps' personnel and, although thought appropriate to these three projects, should not be construed applicable to other situations.

CASE STUDIES:

Demonstration to Ohio River Lock 53 sheet piling: In 1986, severe corrosion of sheet piling at Lock 53 was evident, much more than that at nearby Lock 52, which was constructed 11 years earlier. In addition to questions regarding possible causes for the high corrosion rate, the structural integrity and safety of the sheet piling were investigated by the Corps. Decision analysis could be applied with the use of prior information and estimated probabilities to determine an optimal course of action and the economical feasibility of NDE.

The corrosion at Lock 53 was subjectively estimated to be 8 to 10 percent of steel thickness. Assuming a steel loss of 9 percent gives an estimated corrosion, E = 0.0337 in. For the 6 years since construction, this loss translates to an annual rate of 0.00562 in./year. Projecting this rate over a standard 50-year expected life, expected corrosion is computed as 0.28 in. From the previous reliability estimation (Figure 4), this amount of corrosion provides a reliability of only 0.05; thus probability of failure is 0.95. It appears that rehabilitation is indicated, based on the subjective corrosion estimate. The engineer has the option to buy additional information by performing NDE, specifically ultrasonic testing. Costs of ultrasonic testing, lock rehabilitation, and lock failure must be estimated in order to evaluate the economic aspects. These costs are taken as:

Ultrasonic test	(-)2,500
Lock rehabilitation (normal)	(-)5,000,000
Lock rehabilitation (emergency)	(-)7,500,000

This estimate assumes that emergency rehabilitation after a failure will require short-term emergency repairs and preclude competitive bidding, resulting in a 50-percent increase in cost.

The ultrasonic test does not provide perfect results, and the likelihood of the test's predicting the true state of nature is taken as:

	True State of Nature, t	
Test Result, t'	$t \geq t_{min}$	$t < t_{min}$
$t' \geq t_{min}$	0.94	0.02
Indefinite	0.04	0.04
$t' < t_{min}$	0.02	0.94

In general, the test has a 94-percent probability of predicting the true state of nature, a 2-percent chance of predicting the converse, and a 4-percent chance of providing indefinite results.

The decision analysis is illustrated in Figure 5, indicating the choice of experiments (none or ultrasonic), possible outcomes of the experiment (OK, indefinite, or no good (NG)), actions (do nothing, rehabilitate), true states of nature (lock OK, lock fails), and the expected utilities of each consequence. In order to simplify the numbers, utilities are shown as costs per 1,000. The analysis proceeds from right to left, i.e. from expected utilities to optimal choice of experiment in the following manner:

a. Compute posterior probabilities for each consequence.

b. Compute expected cost of each action by addition of the product of each utility and its associated probability.

c. Determine optimal action choice (minimum cost), and thus, the expected cost associated with experiment outcome.

d. Compute posterior probability of experiment outcome.

e. Compute utility of experiment by addition of the product of each outcome utility and its associated probability.

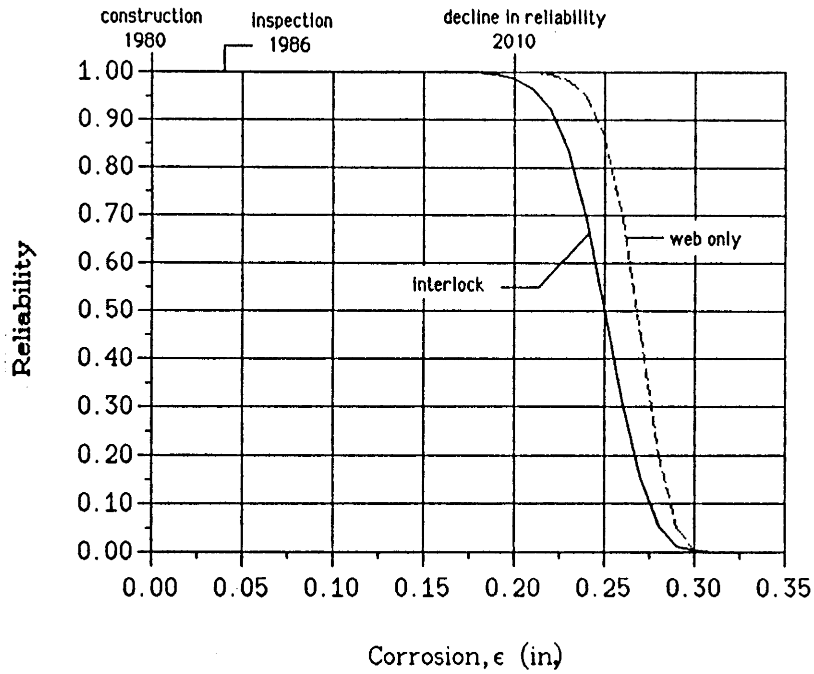

Figure 4. Reliability of temporary lock 53

f. Determine optimal experiment choice by selecting experiment with maximum utility (minimum cost).

On the top branch of the decision analysis (no tests), if the engineer does nothing, there is a 95-percent probability that the utility will be (-)7,500 and a 5-percent probability of no cost. Thus, the expected utility of doing nothing is (-)7,125. If the engineer decides to rehabilitate the lock, he incurs the direct cost of rehabilitation (-)5,000 with a 99-percent probability of lock functioning and a 1-percent chance that the lock will not function, yielding an expected utility of rehabilitation of (-)5,075. Thus, without further information (tests), the engineer should expect to rehabilitate the lock for optimum utility (-)5,075.

If the engineer performs ultrasonic tests, he may expect the three possible outcomes shown: steel thickness is OK, steel thickness is NG, or indefinite results. Having determined the outcome of the tests, his options (actions) are as below; i.e., he must decide whether to do nothing or to rehabilitate the lock. Looking at the branch of the analysis where the tests predict that the steel thickness is OK, the expected utilities are calculated as before, but they now incorporate the new information and costs of the test. The cost of the testing (-)2.5 is now combined with the previous utilities, as shown. The posterior probabilities for the true states of nature (lock OK, lock NG) are computed according to Bayes' theorem. For the action "do nothing" and state "lock OK," the posterior probability is computed as:

$$P''(\text{lock OK}) = \frac{P[t' \text{ OK} \mid t \text{ OK}]\ P'[\text{lock OK}]}{\Sigma P[\text{test}]\ P'[\text{state}]} \qquad (2)$$

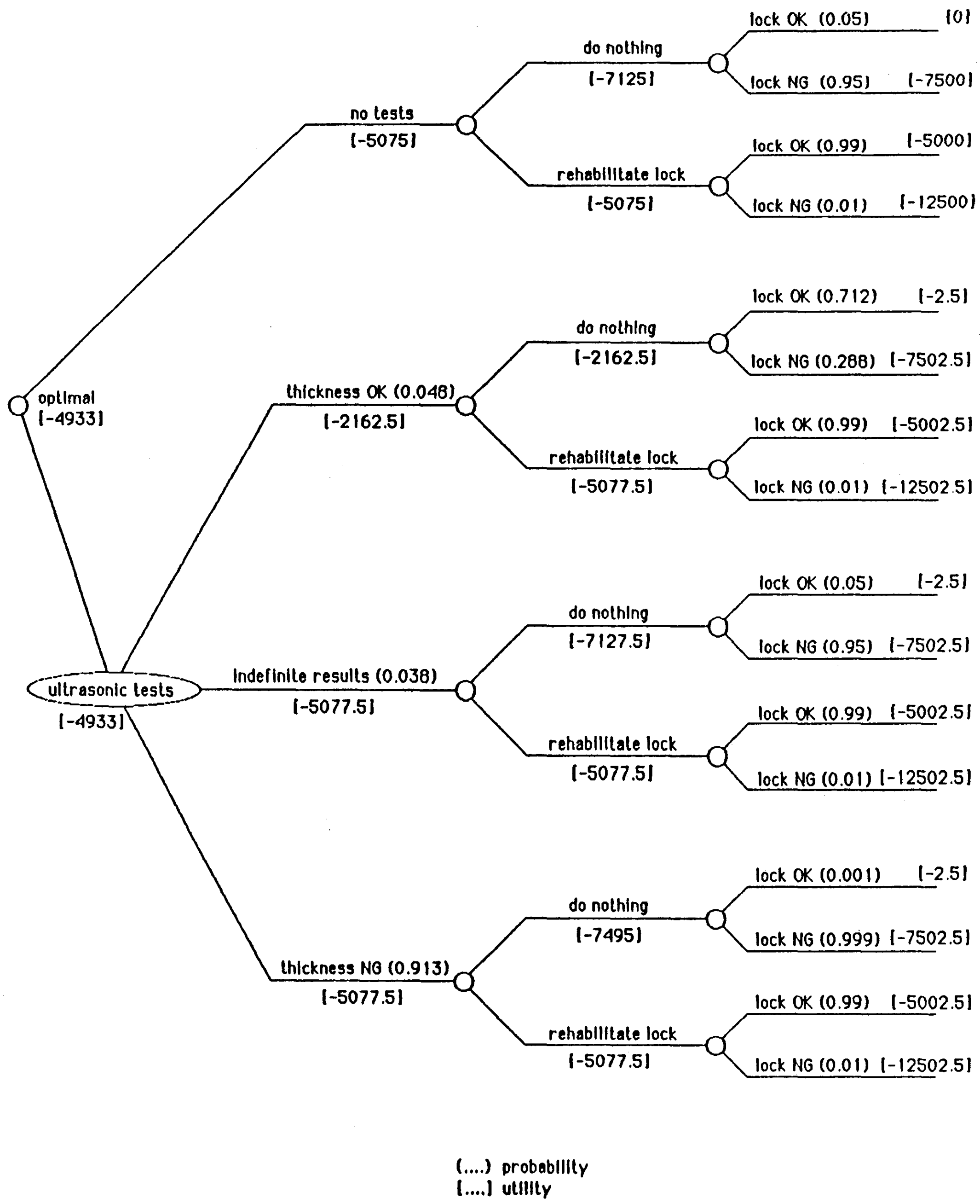

Figure 5. Ohio River Lock 53 decision analysis

$$= \frac{(0.94)(0.05)}{[(0.94)(0.05) + (0.02)(0.95)]}$$

$$= 0.712$$

For the action "do nothing" and state "lock NG," the posterior probability is computed as:

$$P''(\text{lock NG}) = \frac{P[t' \text{ OK} \mid t \text{ NG}]\ P'[\text{lock NG}]}{\Sigma P[\text{test}]\ P'[\text{state}]} \tag{3}$$

$$= \frac{(0.02)(0.95)}{[(0.94)(0.05) + 0.02)(0.95)]}$$

$$= 0.288$$

In each case, the prior state probability is modified by the likelihood of the test's predicting that state. Notice that now, if the tests predict an acceptable thickness, the probability of the lock's functioning is increased dramatically. The probabilities for state do not change for the case in which the lock is rehabilitated, since the test has no influence on this state. Given these posterior probabilities, the expected values of utility for each action are computed as before, yielding (-)2162.5 if nothing is done and (-)5077.5 for rehabilitation. The rehabilitation utility is obviously the sum of the previous value and the cost of the test, since the test provides no new information for this action. Therefore, the optimal utility if the tests predict adequate thickness is (-)2162.5, the expected cost of "doing nothing."

Calculations for the other possible outcomes of the test are similar, yielding the expected utilities shown in the figure. Posterior probabilities for each outcome are computed with the use of Bayes' theorem, using prior probabilities of state and the likelihood of accurate test prediction. Prior probabilities for adequate and inadequate thickness are 0.05 and 0.95, respectively, and the probability of the test's predicting the true state of nature is 0.94. Thus, the posterior probability for adequate thickness is:

$$P''(\text{t OK}) = \frac{(0.94\ (0.05)}{[(0.94)(0.05) + (0.04)(0.04) + (0.94)(0.95)]} \tag{4}$$

$$= 0.048$$

In this case, the prior probability of indefinite results was assumed to be the same as the likelihood of test prediction, a factor accounted for in the normalization term (denominator). Posterior probabilities are computed similarly for inadequate thickness and indefinite results yielding the values shown in the figure. The utility of performing the ultrasonic test is computed as before by summing the products of the outcome utilities with their respective probabilities, yielding (-)4,933.

Comparing the utilities of the experiment choices indicates that testing provides optimal utility, i.e. less expected cost than not performing the tests. This decision analysis, by itself, predicts that the possible advantages of obtaining additional information (tests) outweigh the associated costs of the tests. It should be noted that the decision analysis cannot predict the outcome of the test. For example, if the tests indicate inadequate thickness (91.3-percent probability), rehabilitation is indicated, and the utility is (-)5077.5. This does not mean that the engineer should not have performed the tests. Indeed, he now has much more confidence and justification for lock rehabilitation. On the other hand, if the tests indicate adequate thickness (4.8-percent probability), the probability of lock survival is significantly enhanced to 71.2 percent, and he can postpone the decision to rehabilitate at least until conditions (corrosion) require another decision. At this latter time he will have even more information to incorporate into the decision analysis.

Thus, the preposterior decision analysis has incorporated prior information and expected new information along with their estimated costs and utilities to provide the engineer another tool to evaluate his options and make an economically justifiable decision.

Demonstration to Emsworth Dam Spillway Gate: In 1971, the first periodic inspection of the lift gates at Emsworth Dam indicated "severe corrosion" and prompted the Corps to evaluate the structural integrity and safety of the gate. Assuming that "severe corrosion" can be subjectively quantified as about 33 percent loss of steel thickness on the critical brace identified in Figure 6, the average corrosion rate (over 35 years) is computed to be 0.0054 in./year. Projecting this rate over a standard design life of 50 years predicts a total corrosion of 0.268 in., indicating a reliability of 0.92 based on the estimation shown in Figure 7. This high probability, taken with the costs of rehabilitating the gate, tends to indicate that nothing should be done at this time. However, the engineer has the option to buy additional information by performing tests. For example, he could request detailed visual examination with physical measurements (micrometer or thickness gages) or ultrasonic testing.

The decision analysis shown in Figure 8 includes these two experiment options and is based on the following data (for one gate):

Cost of detailed visual testing	(-)2,000
Cost of rehabilitation (normal)	(-)2,000,000
Cost of rehabilitation (emergency)	(-)3,000,000

This estimate assumes that emergency rehabilitation after a failure will require short-term emergency repairs and preclude competitive bidding, resulting in a 50-percent increase in cost.

The visual evaluation does not not provide perfect results, and the likelihood of the test's predicting the true state of nature is taken as:

	True State of Nature, t	
Test Result, t'	$t \geq t_{min}$	$t < t_{min}$
$t' \geq t_{min}$	0.85	0.05
Indefinite	0.10	0.10
$t' < t_{min}$	0.05	0.85

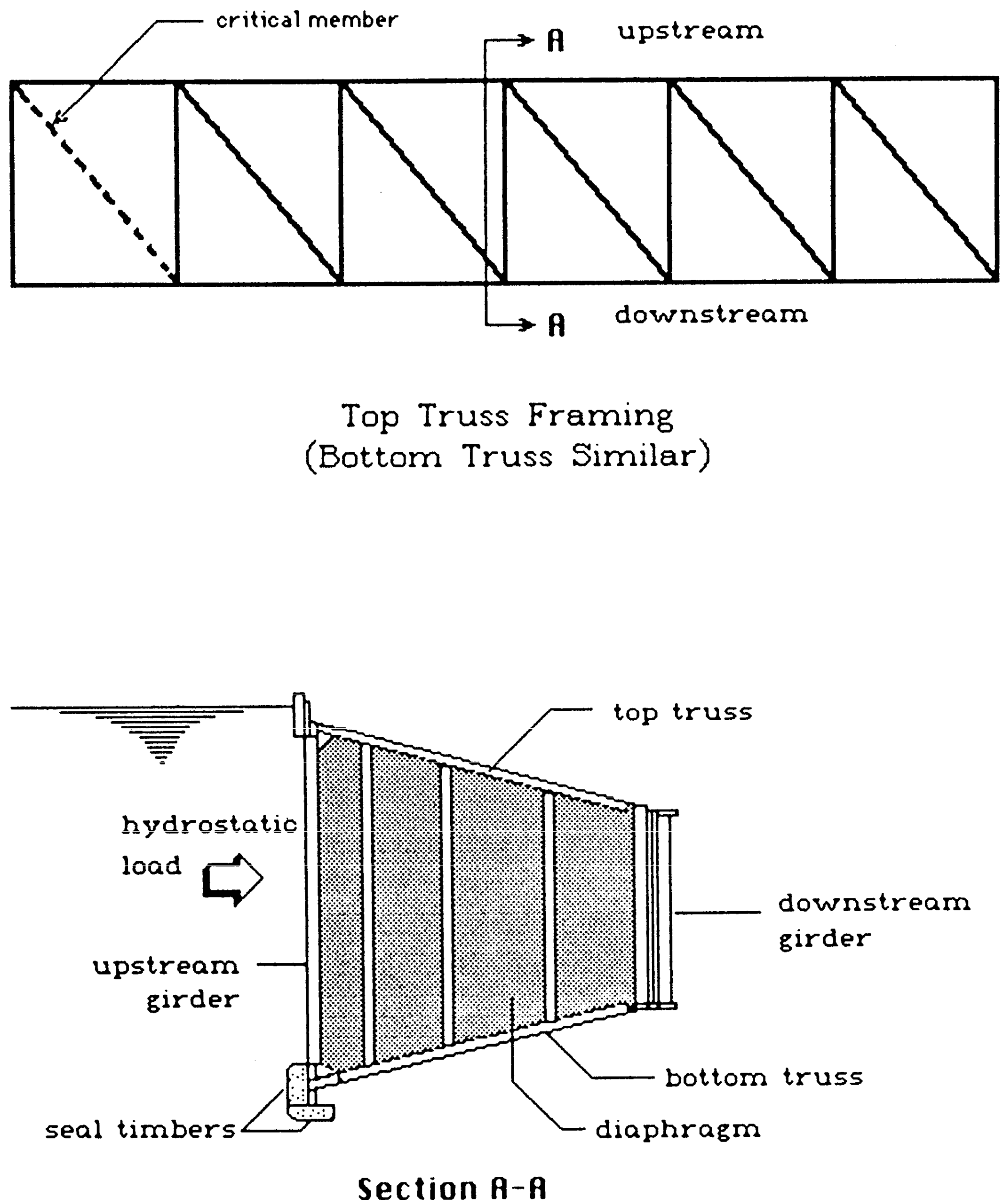

Figure 6. Emsworth vertical lift gate

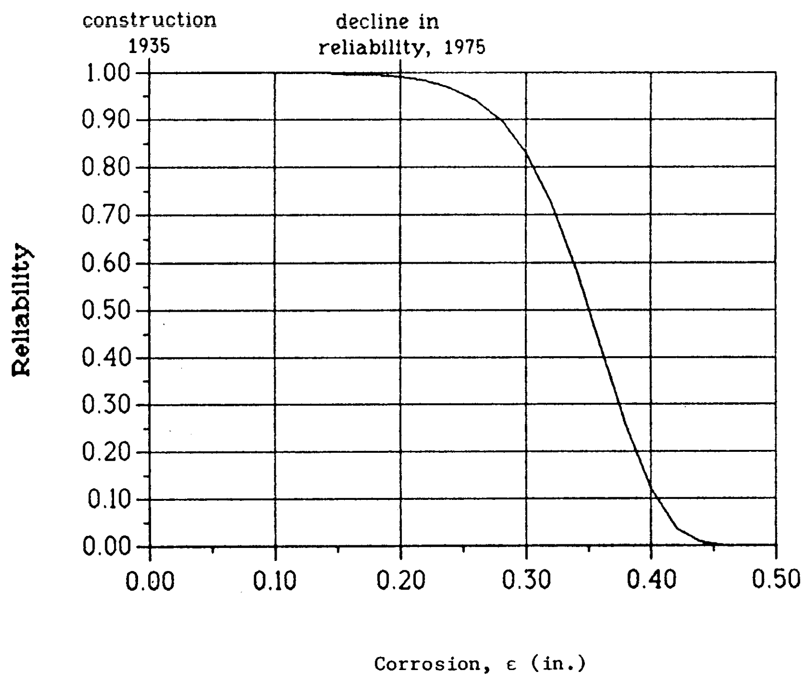

Figure 7. Reliability of Emsworth lift gate

Data for ultrasonic testing are taken as in the previous application.

The decision analysis using these data is illustrated in Figure 8 and includes three choices of experiment: no tests, visual, and ultrasonic. For each experiment choice, the outcomes, actions, and states of nature are the same as in the previous application with the utilities and probabilities based on the relevant data for Emsworth. Calculation of posterior probabilities and expected utilities proceed as described previously, yielding the results shown. Even though the gate has a high prior probability of survival if nothing is done, decision analysis indicates that utility is improved if either test is conducted. According to the analysis, visual testing is more optimal than ultrasonic testing, even though ultrasonic is assumed to be more accurate. The decision analysis incorporates the accuracy of the test method without favoring the results of a more accurate test. For example, looking at the case for each test in which inadequate thickness is predicted and nothing is done, the posterior probabilities of failure are vastly different for visual and ultrasonic. For this consequence after visual testing, probability of failure is about 59.6 percent, whereas the probability of failure after ultrasonic testing is about 80.3 percent. Since these posterior probabilities are factored into the expected utilities, the expected cost for doing nothing after ultrasonic testing is higher than after visual testing. These values are, of course, logical, since the results of ultrasonic are more likely to predict the true state of nature.

In addition to indicating the optimal choice of experiment, relative closeness of the utilities for visual and ultrasonic could prompt more accurate assessment of utilities and probabilities to determine if the relative utilities are affected.

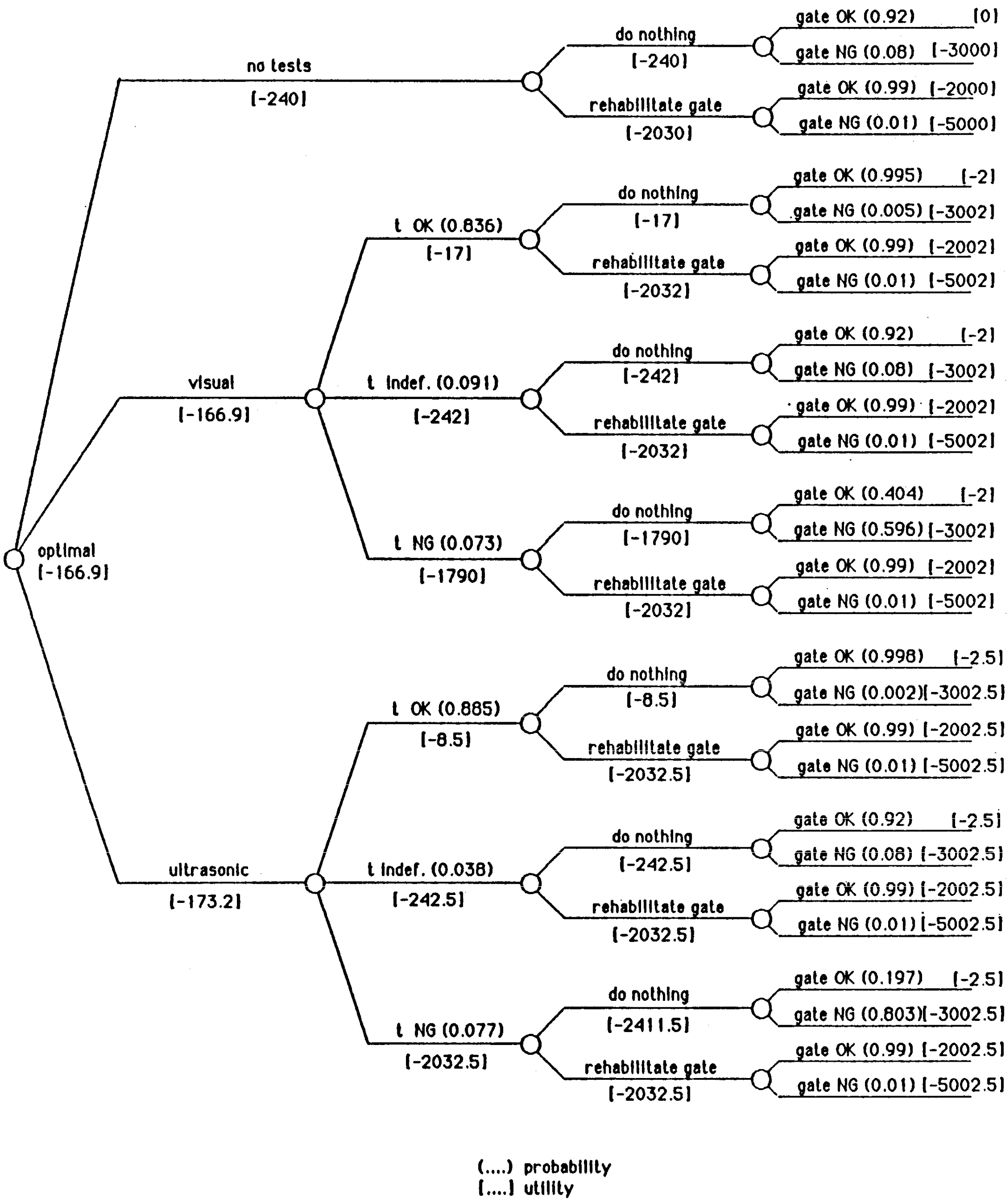

Figure 8. Emsworth Dam gate decision analysis

Demonstration to John Day Lock Gate: During the annual inspection of the downstream lock at John Day Dam in 1982, major cracks were discovered in tension tie members of the bottom six arches. Because of the sporadic and sometimes rapid rate of crack propagation, an annual crack propagation rate cannot be rationally computed to extrapolate and determine reliability at the design life of the structure. Decision analysis can be applied, however, by assuming a future value of reliability for prior probability. For this case, a prior reliability is assumed to be 0.001, since there is a random chance that the cracked gate will function. Two NDE procedures considered for determining crack length, c , are detailed visual and dye penetrant testing. The relevant data used in the analysis are as follows:

Detailed visual test cost	(-)500
Dye penetrant test cost	(-)3,000
Gate rehabilitation (normal)	(-)50,000
Gate rehabilitation (emergency)	(-)2,300,000

This estimate assumes that emergency rehabilitation after a failure not only will require short-term emergency repairs and preclude competitive bidding, but may require replacement of the entire gate and may cause other damages.

The visual test is assumed to have the following likelihoods of the test's predicting the true state of nature:

	True State of Nature, c	
Test Result, c'	$c \geq c_{min}$	$c < c_{min}$
$c' \geq c_{min}$	0.70	0.20
Indefinite	0.10	0.10
$c' < c_{min}$	0.20	0.70

Dye penetrant testing is assumed to have the following likelihoods of the test's predicting the true state of nature:

	True State of Nature, c	
Test Result, c'	$c \geq c_{min}$	$c < c_{min}$
$c' \geq c_{min}$	0.96	0.02
Indefinite	0.02	0.02
$c' < c_{min}$	0.02	0.96

The decision analysis is illustrated in Figure 9 and is computed as in the previous cases. The obvious result of the analysis is that further testing is not optimal and directly increases the cost for either experiment choice by the amount of the test cost. The calculations are completely driven by the extremely low prior reliability, directing repair of the gate regardless of the experiment outcome.

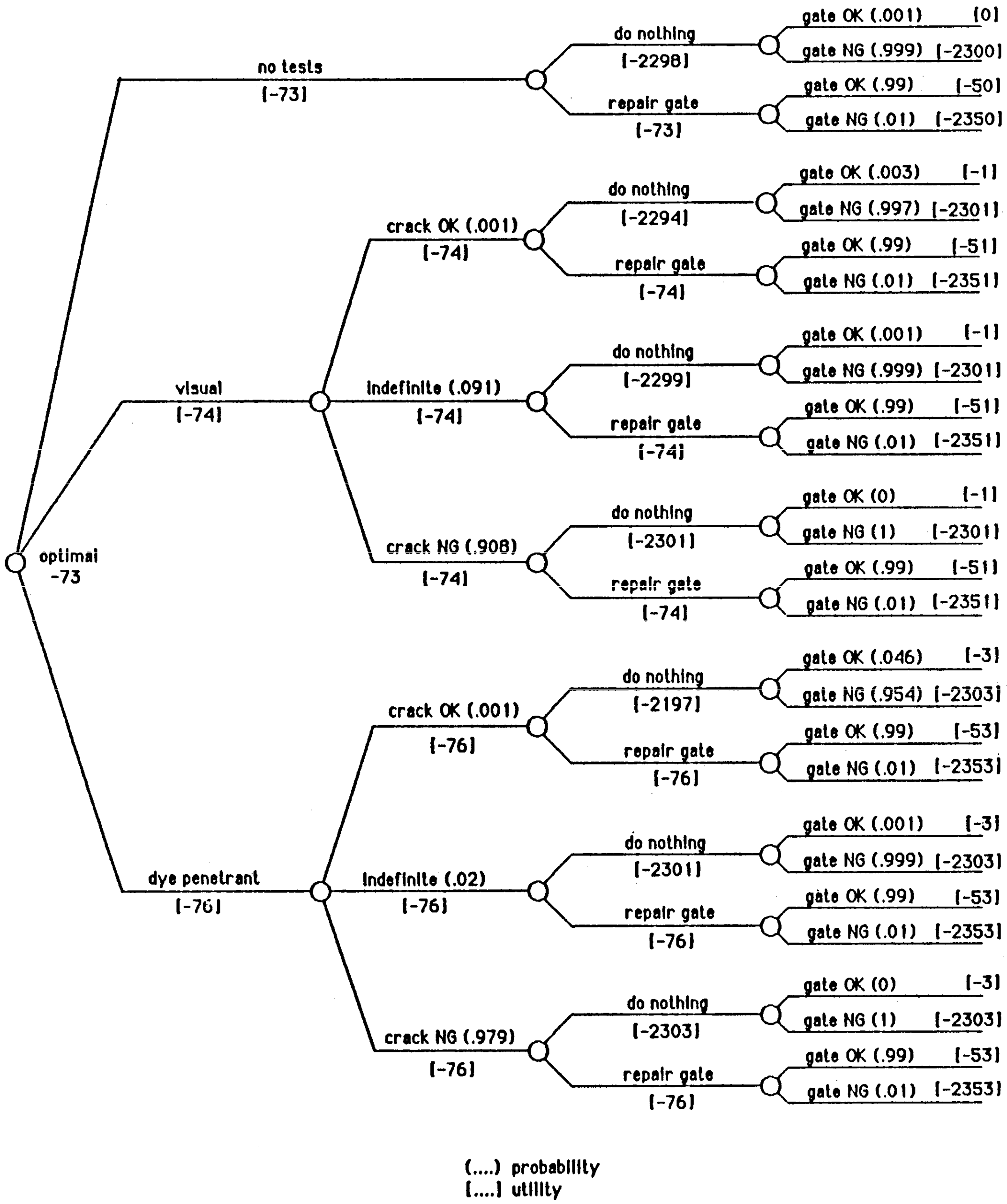

Figure 9. John Day lock gate decision analysis

This result is entirely consistent with engineering judgment and with the actions that were taken at the time. There could be little doubt that the gate should be repaired as soon as possible, given the extent of cracking and the nature of crack propagation. This demonstrates the capability of the decision analysis to predict optimal choice in an extreme case in which there really is no choice.

GENERAL OBSERVATIONS: The selection of NDE methods for Civil Works metal structures should consider the structural reliability, the NDE reliability, the cost of functional loss, and the cost of NDE. Preposterior Bayesian Decision Analysis explicitly accepts this information as quantitative input. The analysis then ranks various NDE alternatives by their expected cost as a function of these inputs. Such a model can thus rationally guide engineering decisions.

REFERENCES:

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b. Hare, B. M., and Hall, W. B. 1986 (Mar). "Probability Models for Nondestructive Evaluation of Deteriorating Metal Structures," Technical Report GE 86-2, University of Illinois, Urbana, IL.